Fundamental Theory
Algebraic Graph Transformation

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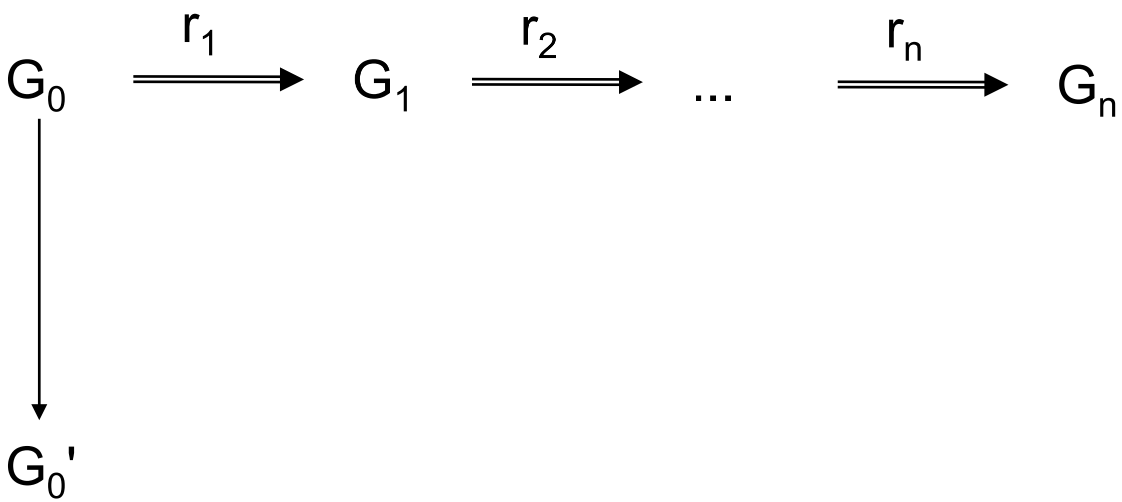
1. Embedding and Extension
2. Independence: Local Church-Rosser Theorems
3. Parallel Rules and Concurrent Rules
4. Conflicts and Critical Pairs

Embedding and Extension

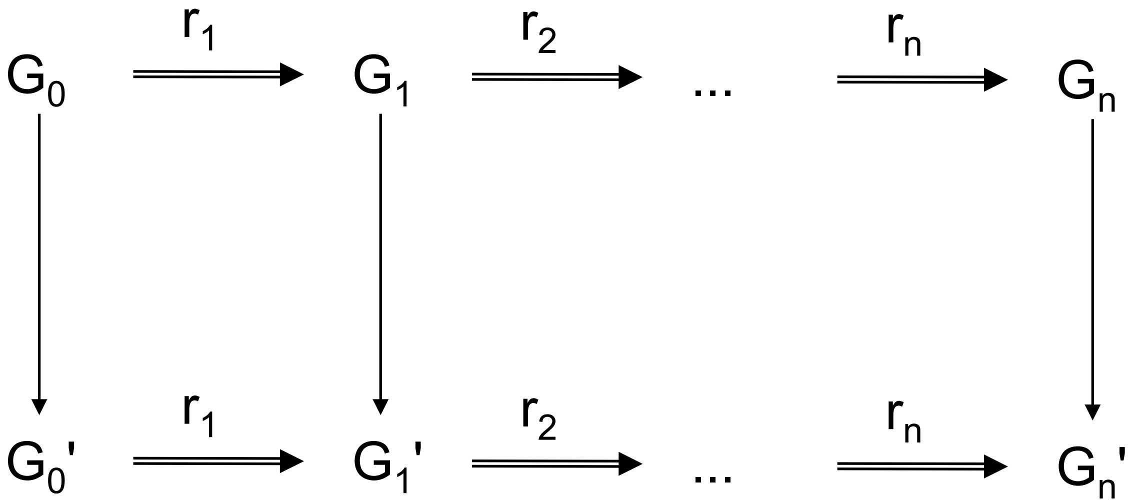
Embedding and Extension Theorems

Given the derivation sequence and a morphism from G_0 to G'_0



Embedding and Extension Theorems

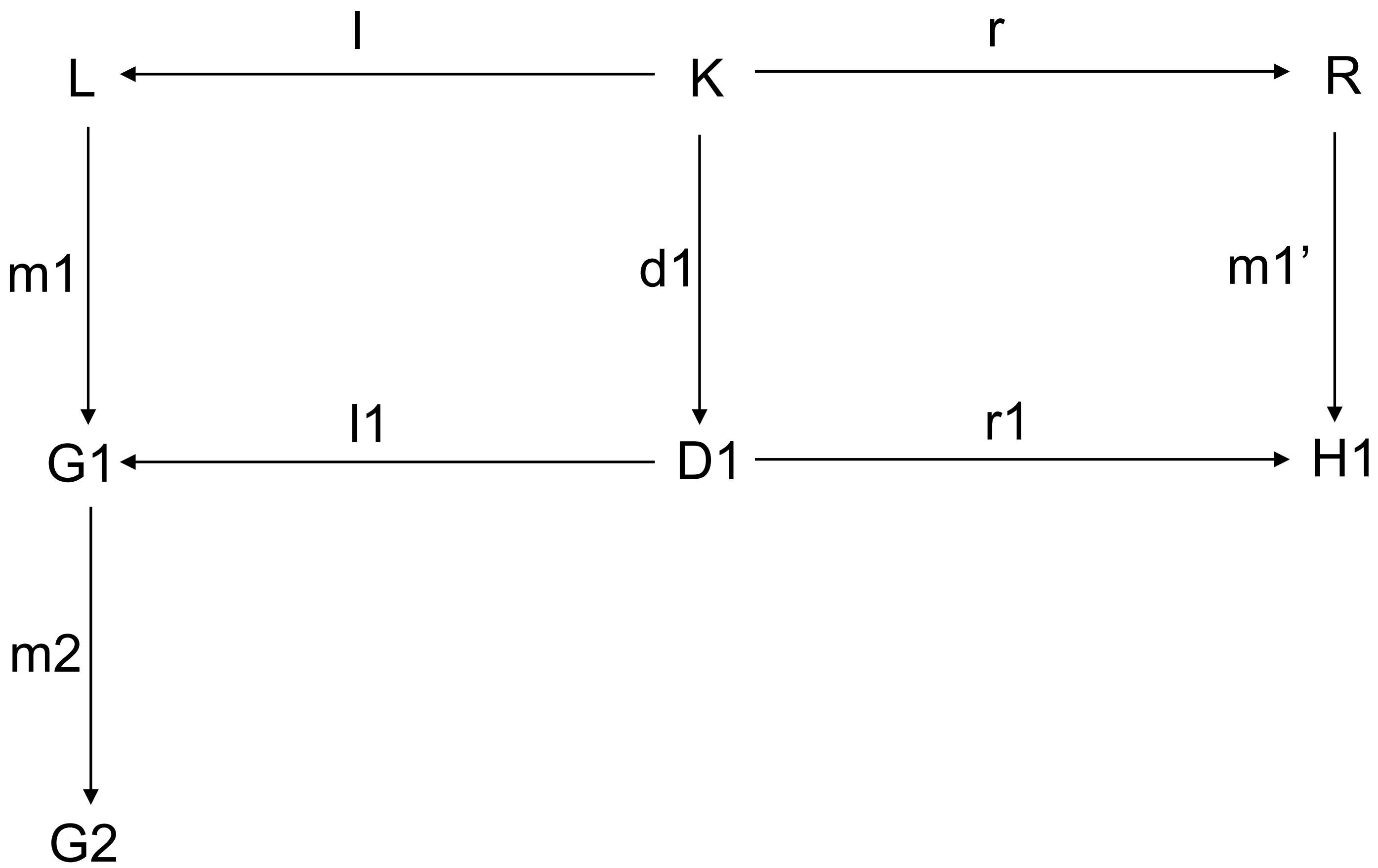
is it always possible to extend it?



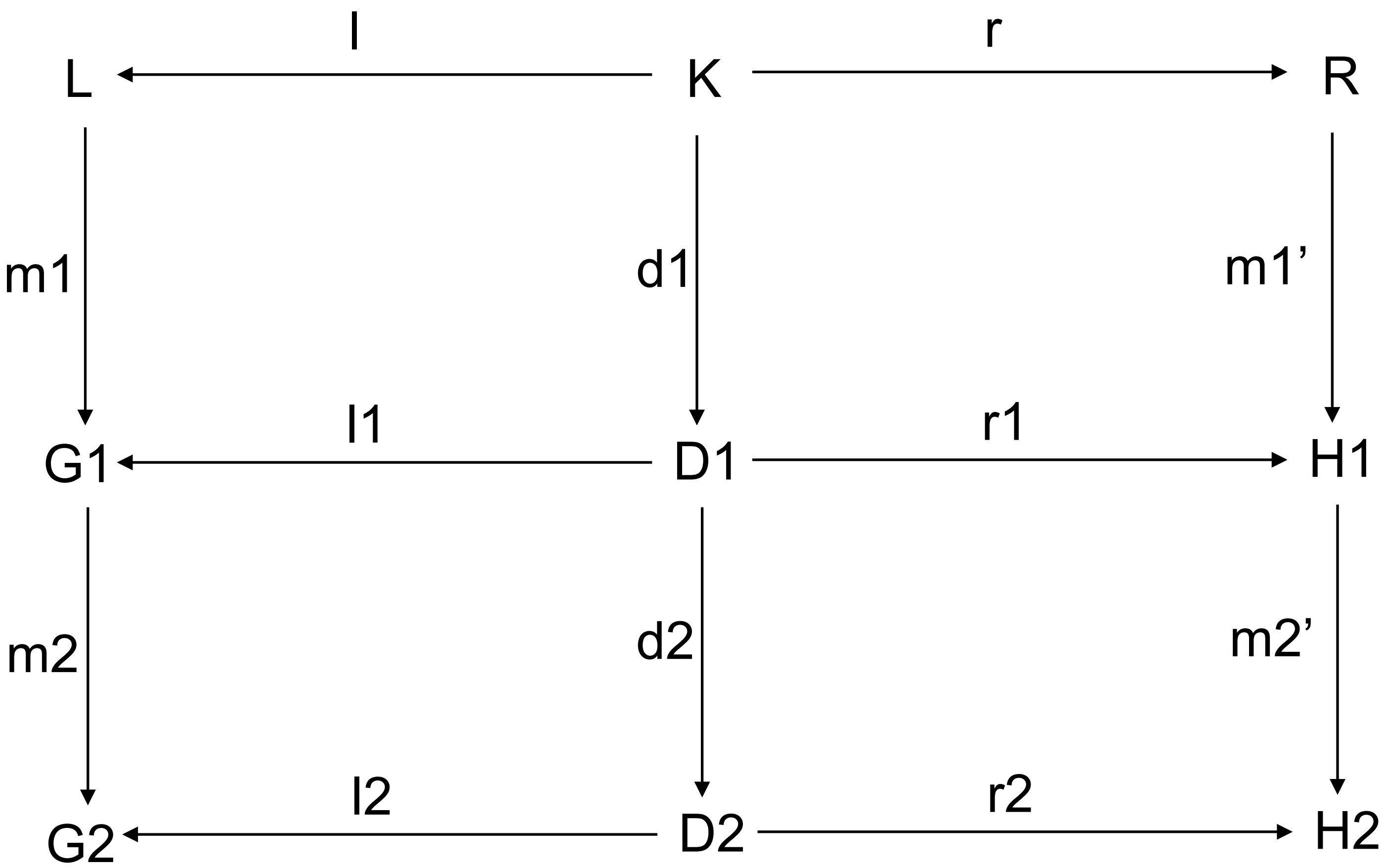
Embedding and Extension Theorems

We can build the extension if and only if m2 is boundary consistent

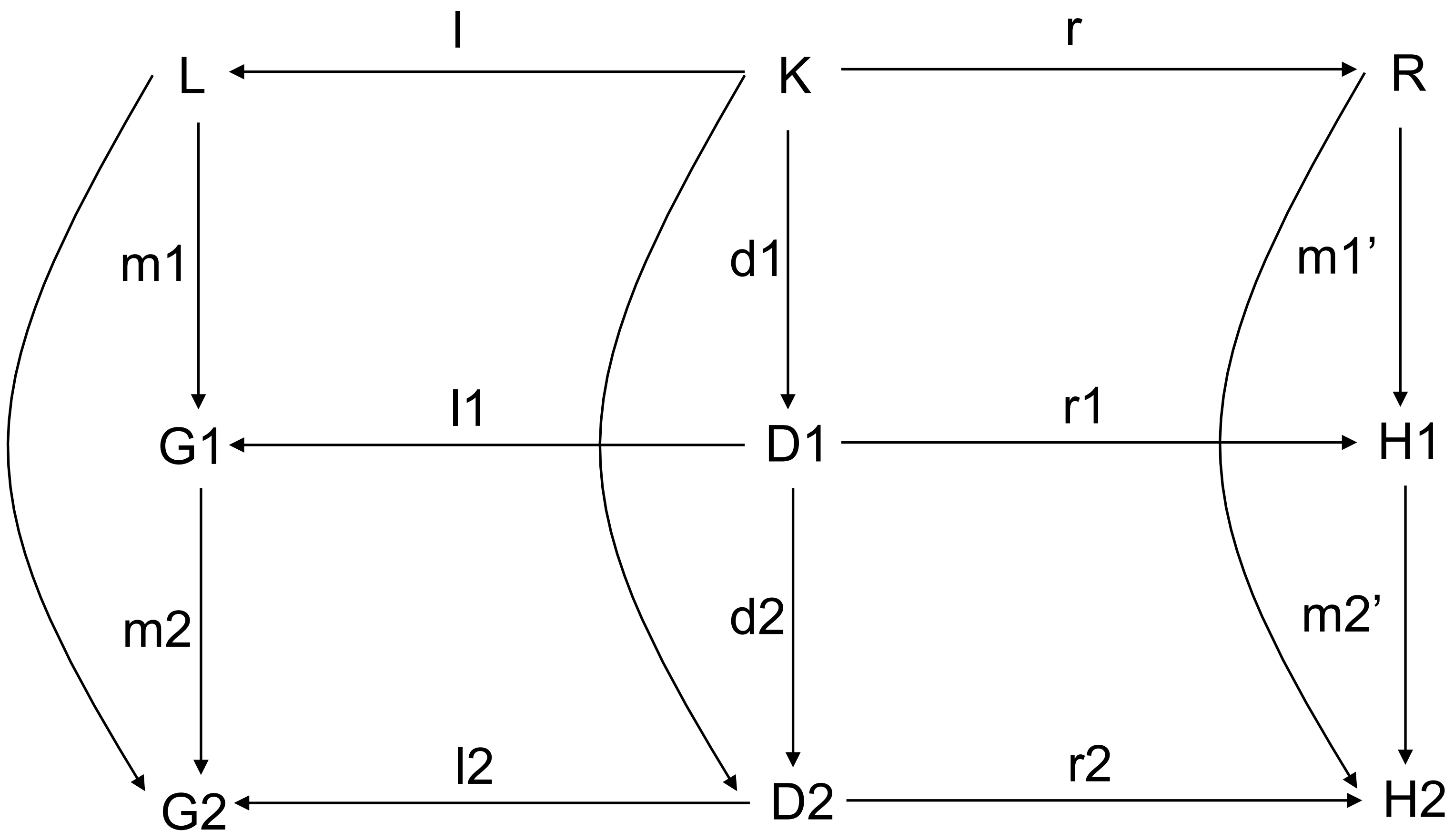
with respect to l1



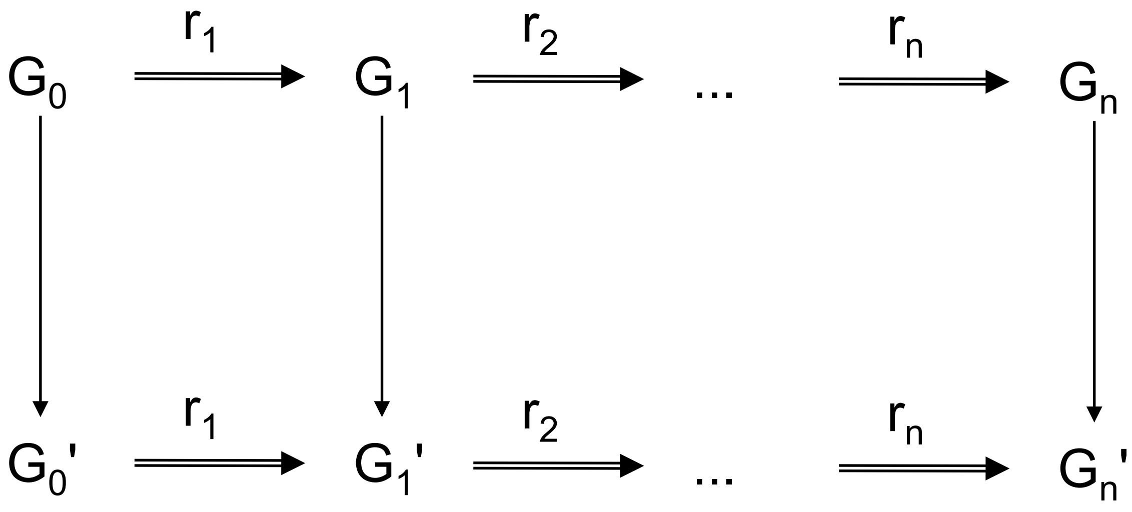
Embedding and Extension Theorems



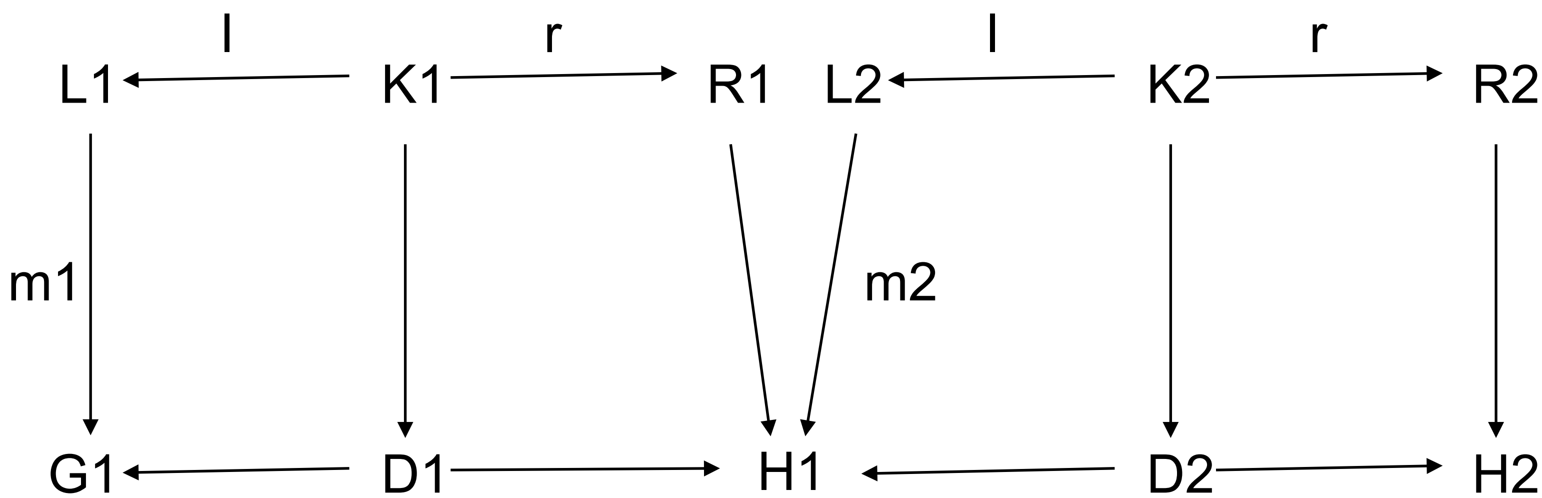
Embedding and Extension Theorems



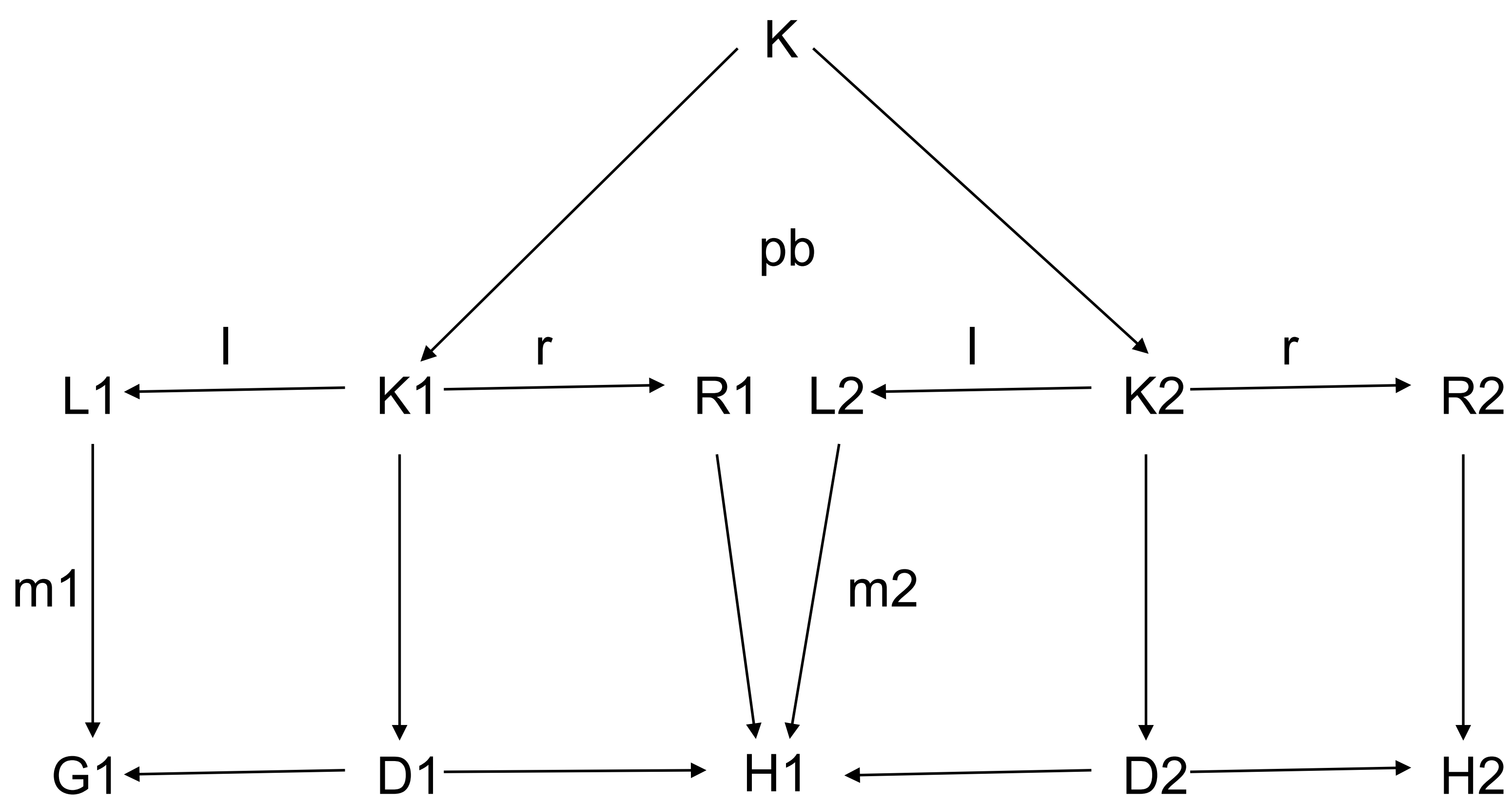
Embedding and Extension Theorems



Derived Rule

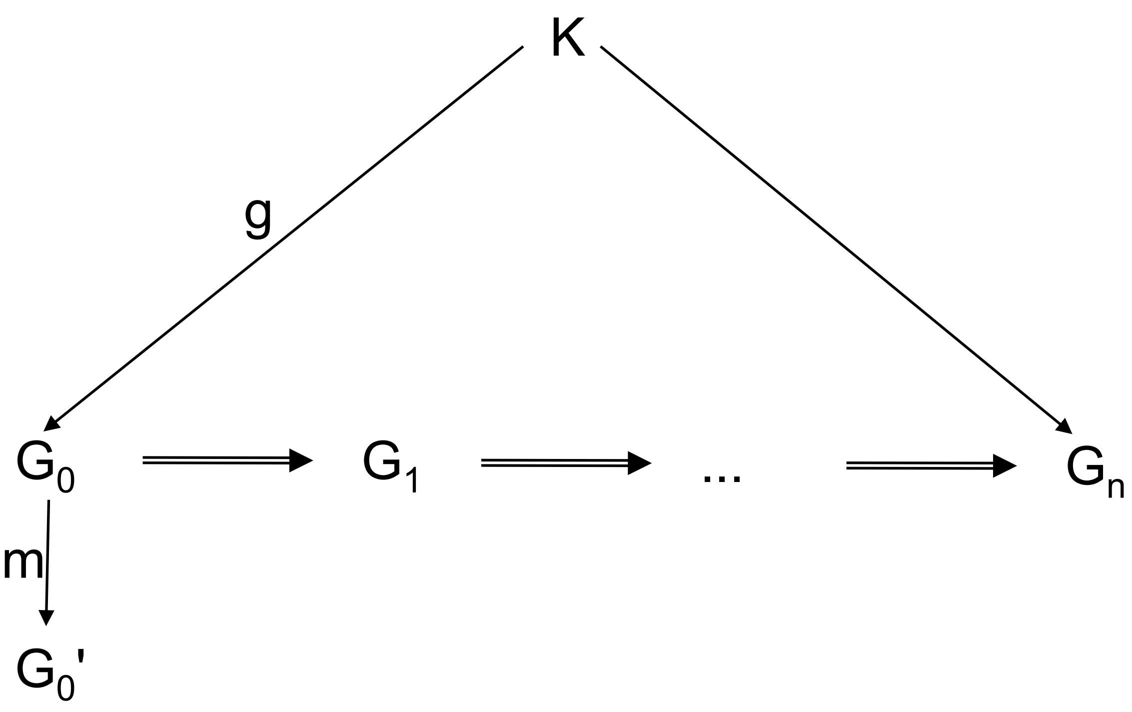


Derived Rule



Embedding and Extension Theorems

Given:

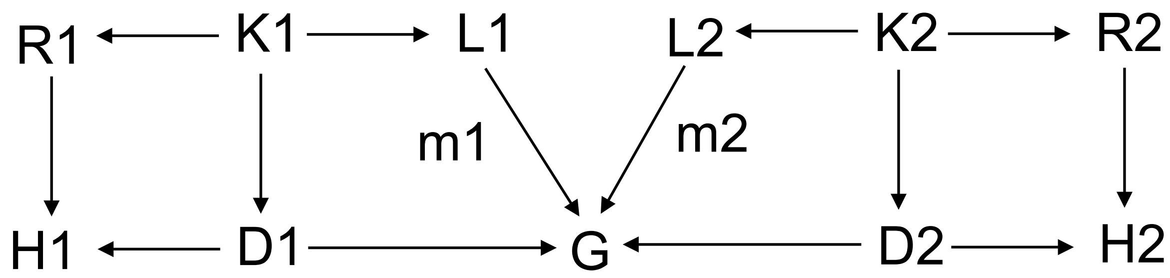


$G_0 \Rightarrow^* G_n$ can be extended to a derivation $G'_0 \Rightarrow^* G'_n$ if and only if m is boundary consistent with respect to g.

Independence: Local Church-Rosser Theorems

The problem

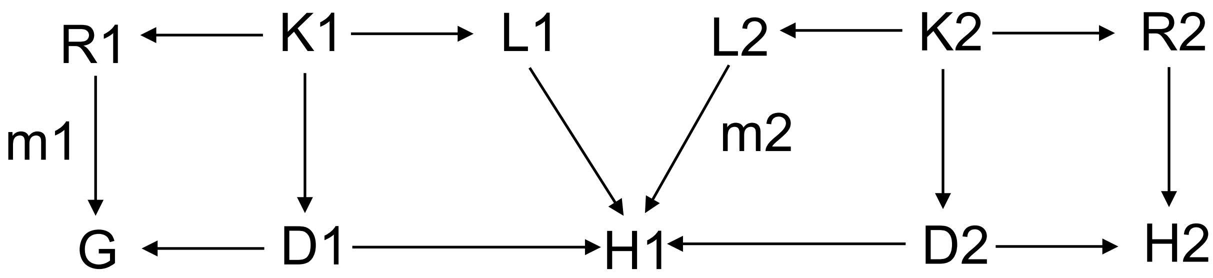
Given two rule applications on the same graph G:



after applying one of them, under which conditions it will be possible to apply the other rule with "the same matching"

The problem

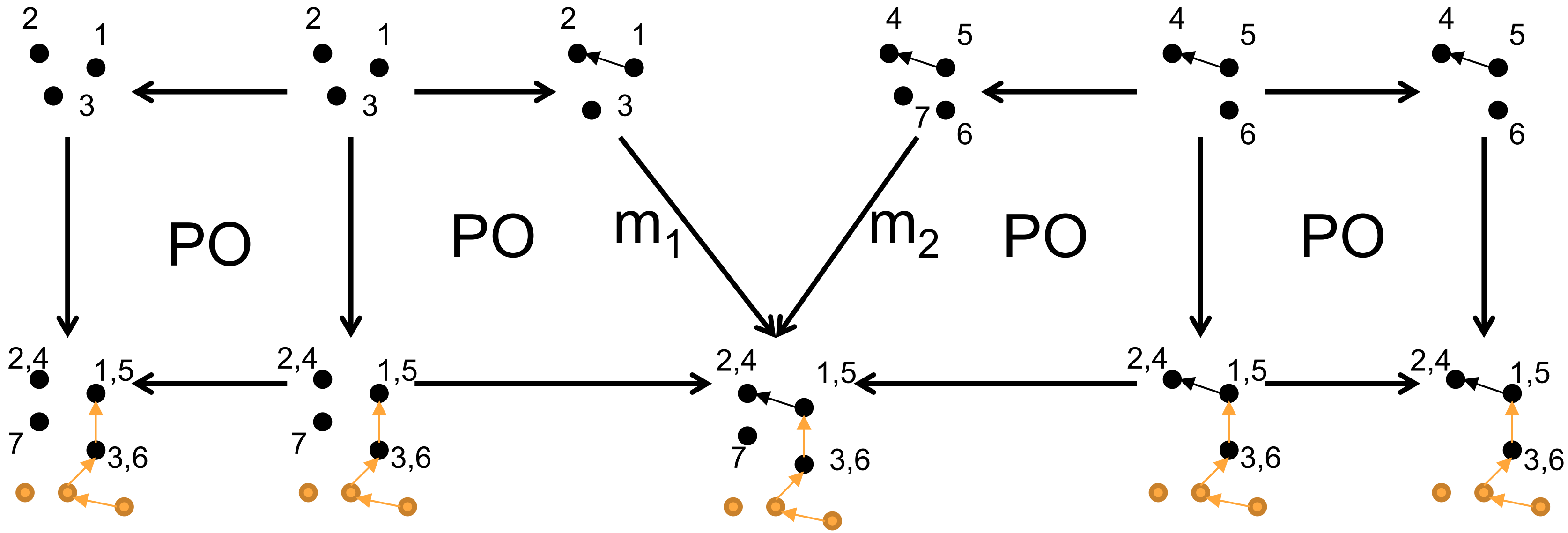
And given two consecutive rule applications on the same graph G:



up to which point the application of the second rule

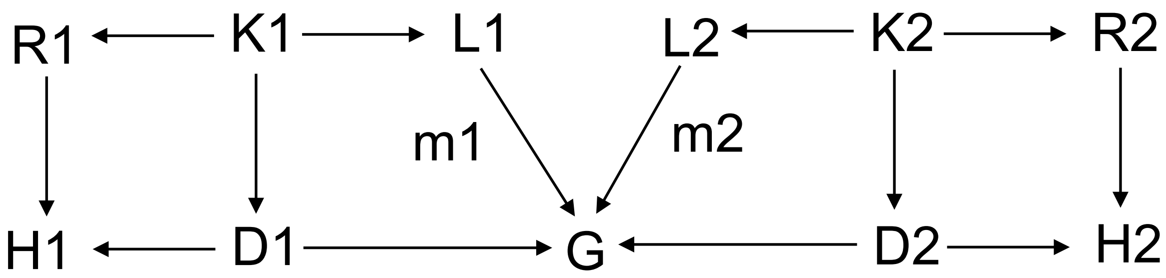
depends or not on the application of the first rule.

Example of non parallel independence



Independence

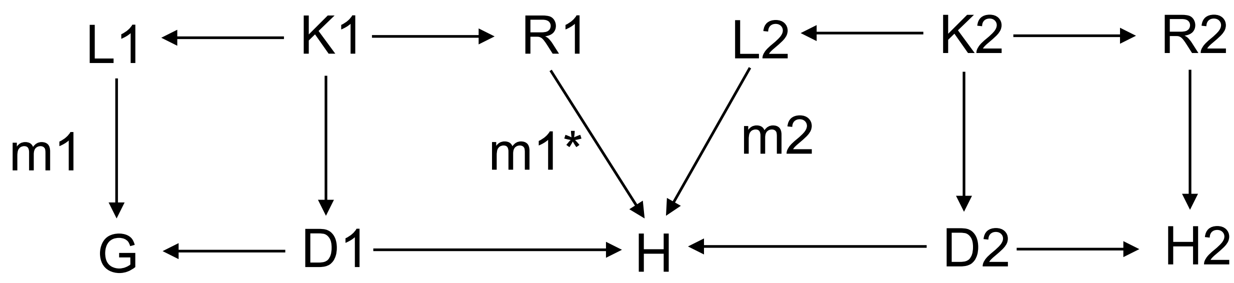
Two rule applications on the same graph G:



are parallel independent if

$$(m1(L1) \cap m2(L2)) \subseteq (m1(K1) \cap m2(K2))$$

Two consecutive rule applications on a graph G:

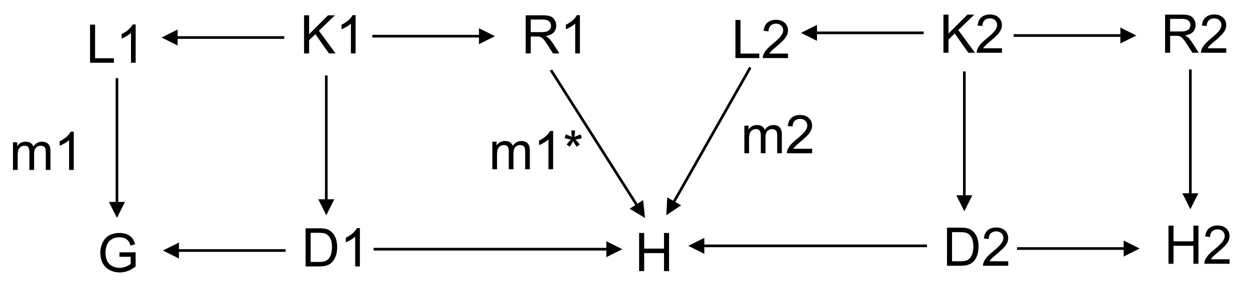


are sequential independent if

$$(m1^*(R1) \cap m2(L2)) \subseteq (m1^*(K1) \cap m2(K2))$$

Independence

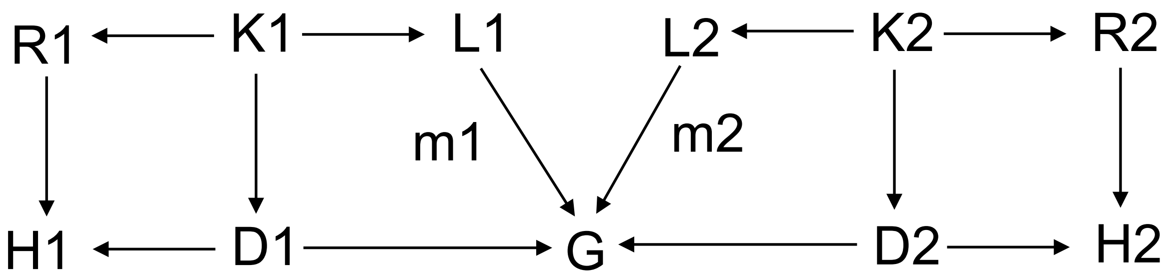
Two rule applications are sequential independent



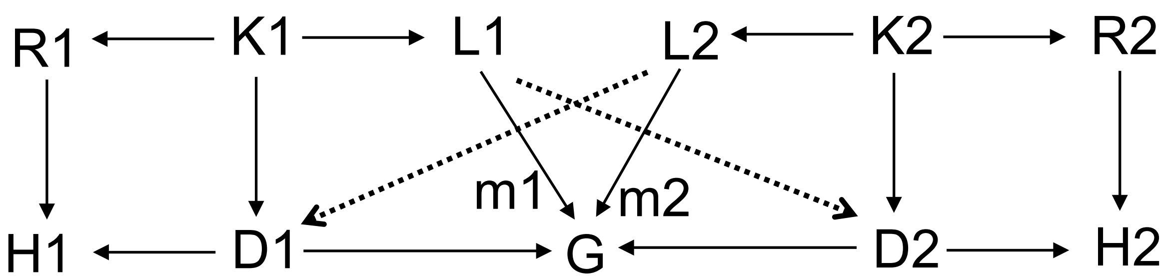
If the reverse application of the first rule and the second application are parallel independent

Parallel Independence

Two rule applications on the same graph G:

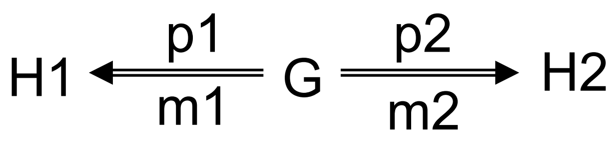


are parallel independent if

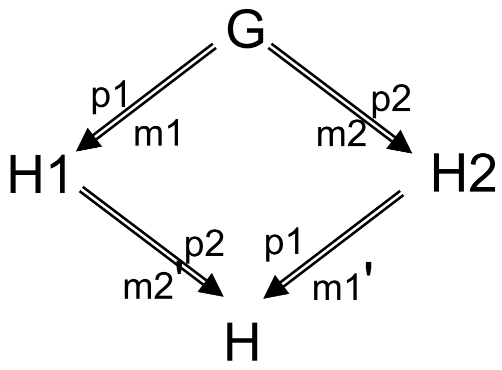


Local Church-Rosser Theorem

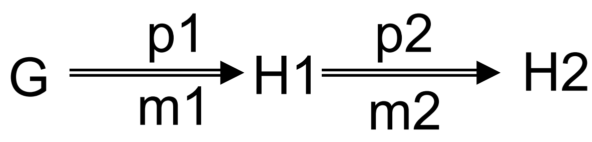
1. Given two parallel independent rule applications of rules p1 and p2 on a graph G:



there are applications of p1 to H2 and of p2 to H1 such that the final result H coincides:



2. Given two sequential independient rule applications of rules p1 and p2 on a graph G:



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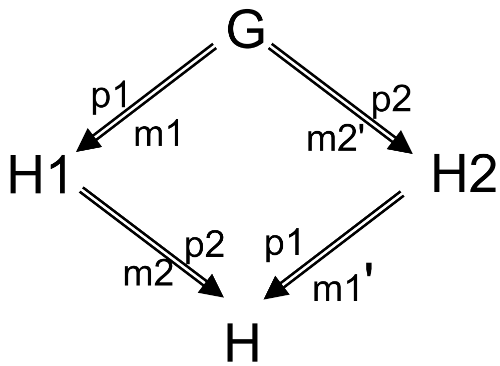


Parallel Rules and Concurrent Rules

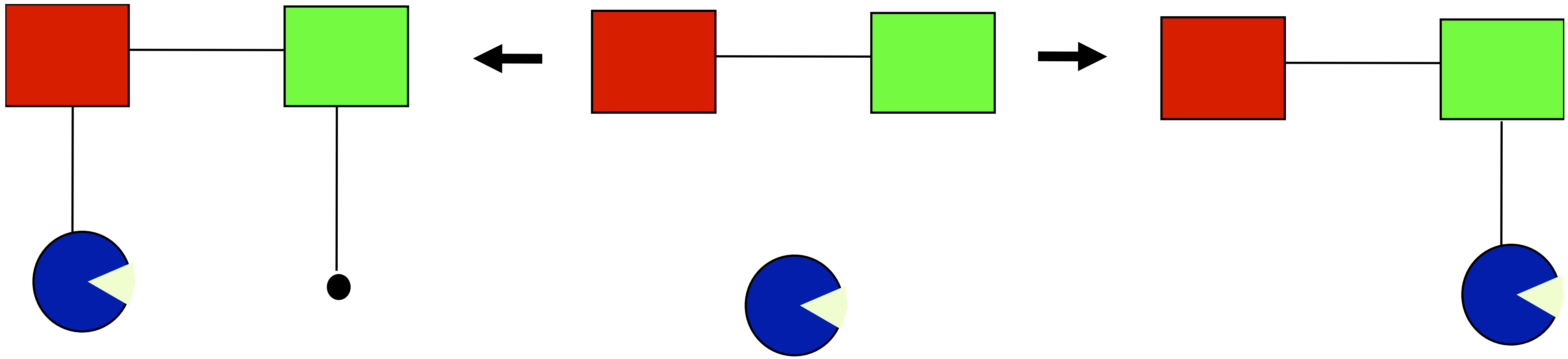
Parallel Composition

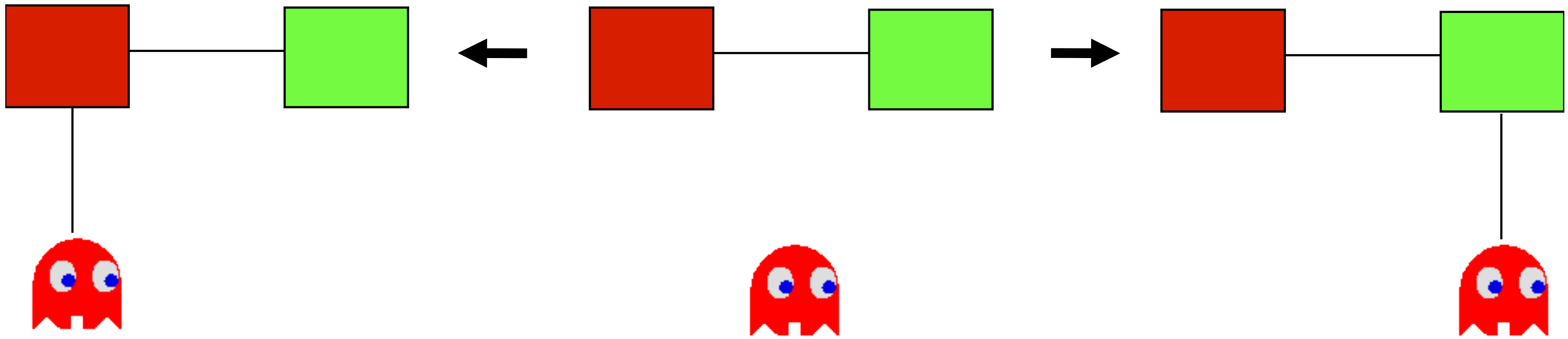
Given rules

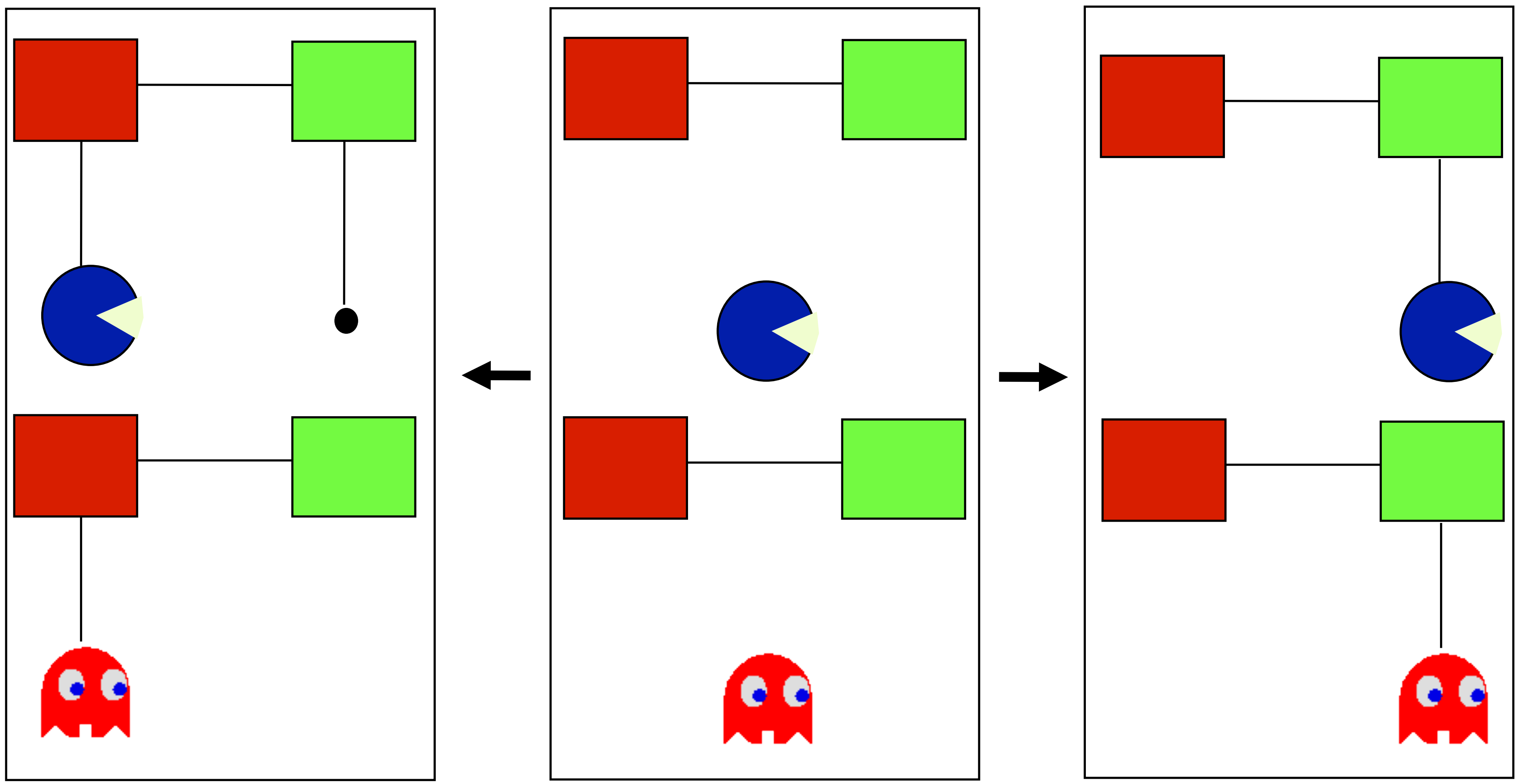
$$L1 \longleftarrow K1 \longrightarrow R1 \qquad L2 \longleftarrow K2 \longrightarrow R2$$

Their parallel composition is the rule:

$$L1+L2 \longleftarrow K1+K2 \longrightarrow R1+R2$$

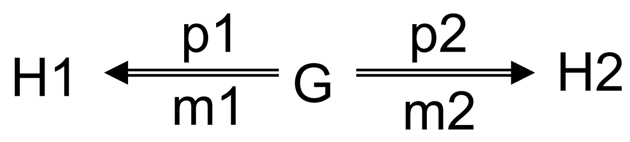




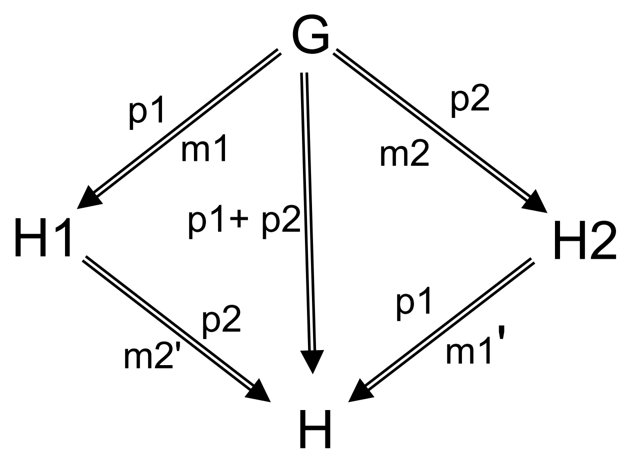


Theorem of Parallelism

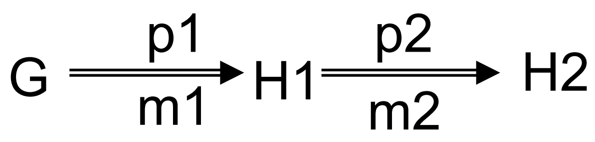
1. Given two parallel independent rule applications of rules p1 and p2 on a graph G:



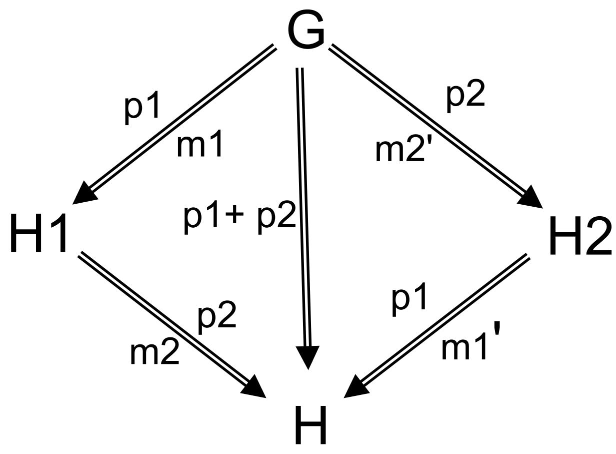
we have:



2. Given two sequential independient rule applications of rules p1 and p2 on a graph G:

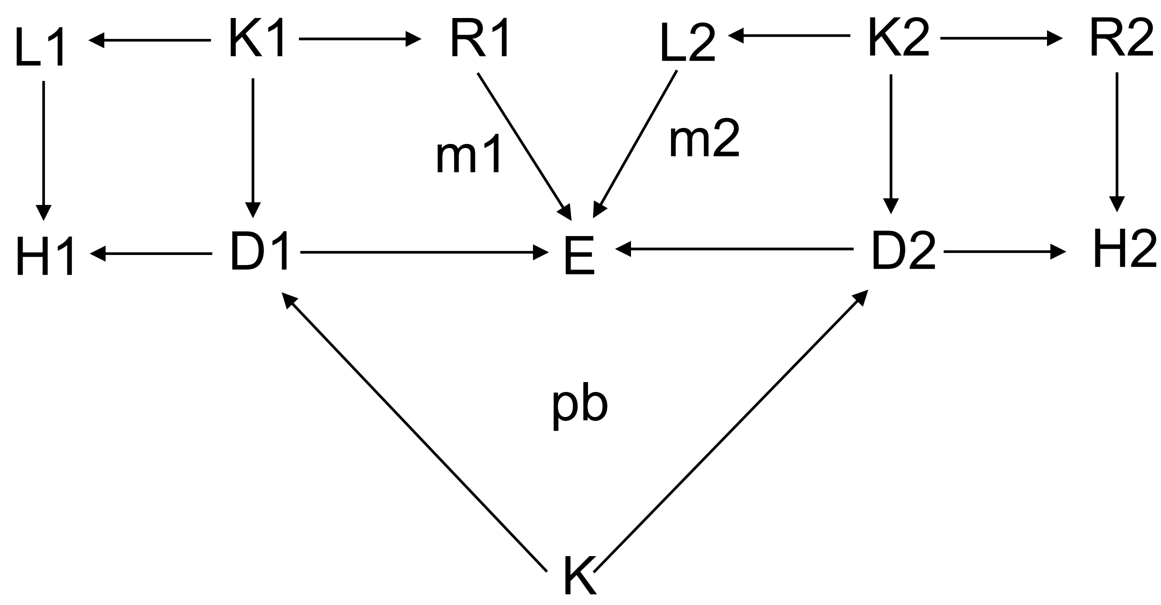


we have:



Concurrent Rules

Given two rules p1 and p2 and jointly epimorphic morphisms m1 and m2 :



$p1*_{(m1,m2)}$ p2 = H1 ← K → H2 is the (m1,m2)-concurrent rule for p1 and p2, if the pushout complements D1 and D2 exist.

Concurrency Theorem

For every transformation G1 $\Rightarrow_{p1}$ G2 $\Rightarrow_{p2}$ G3, there is a concurrent rule p = $p1 *_{(m1,m2)} p2$ such that G1 $\Rightarrow_{p}$ G3.

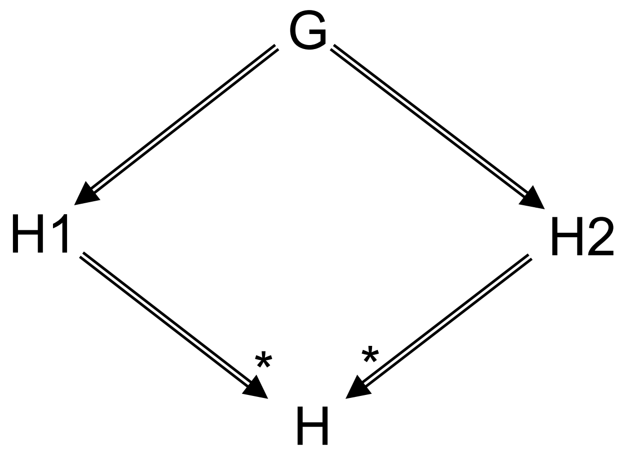
Conflicts and Critical Pairs

Conflicts and Conflict detection

If we use graph transformation to describe a deterministic computation, when we can apply two parallel dependent transformations to a graph G

$$H1 \ {}_{p1,m1}\Leftarrow G \Rightarrow_{p2,m2} H2$$

we have a possible conflict, which is solved if afterwards we can converge to a common result H:



A graph transformation is locally confluent if all conflicts can be solved.

Confluence and Local Confluence

Confluence and local confluence are undecidable for graph transformation systems, even if they are terminating.

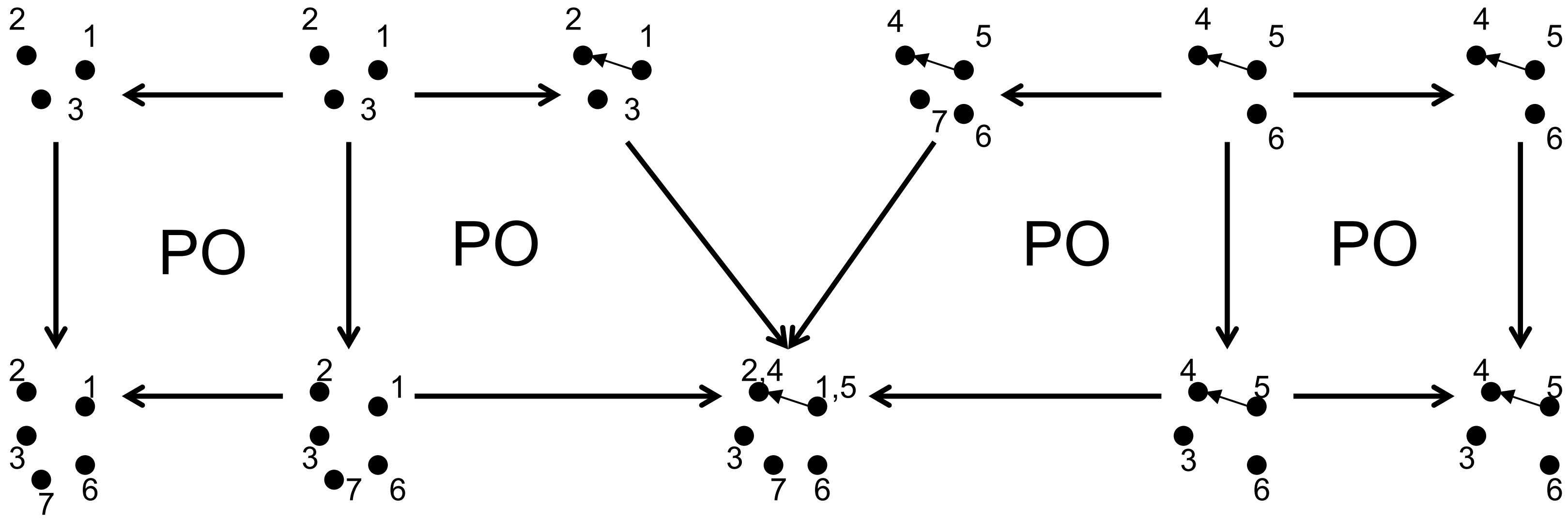
Conflicts and Conflict detection

A critical pair is a minimal conflict: two direct transformations

$$P1 \;_{p1,m1}\!\Leftarrow G \Rightarrow_{p2,m2} P2$$

such that they are not parallel independent and m2: L1 $\rightarrow$ G and m2: L2 $\rightarrow$ G are jointly epimorphic.

Example: a Critical Pair

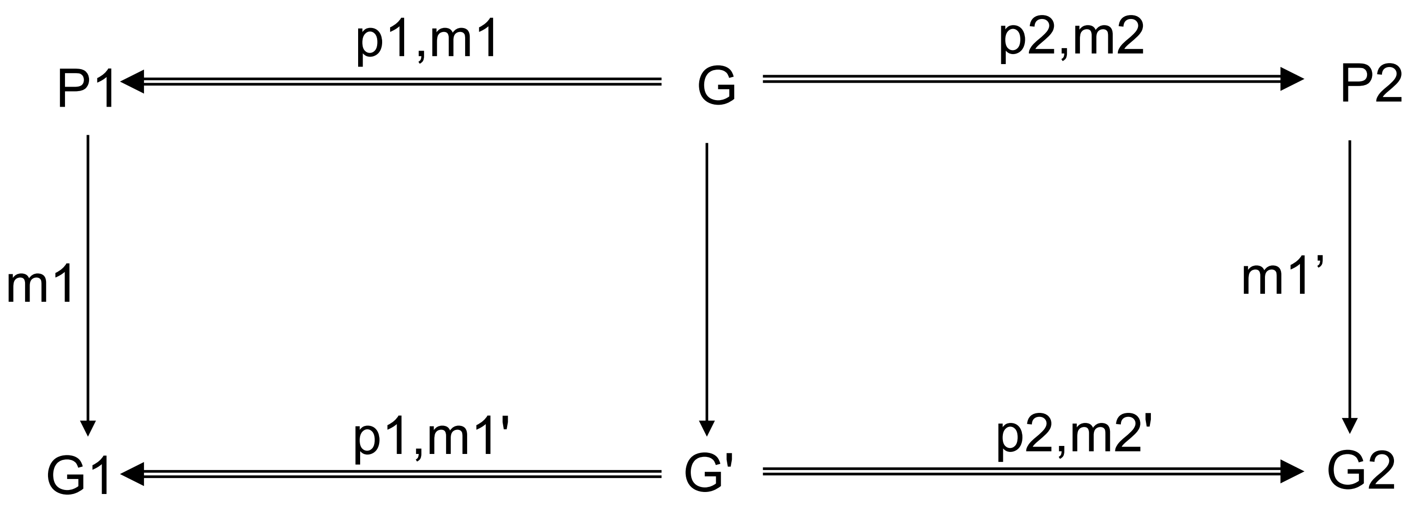


Completeness Theorem

For any two parallel dependent direct transformations:

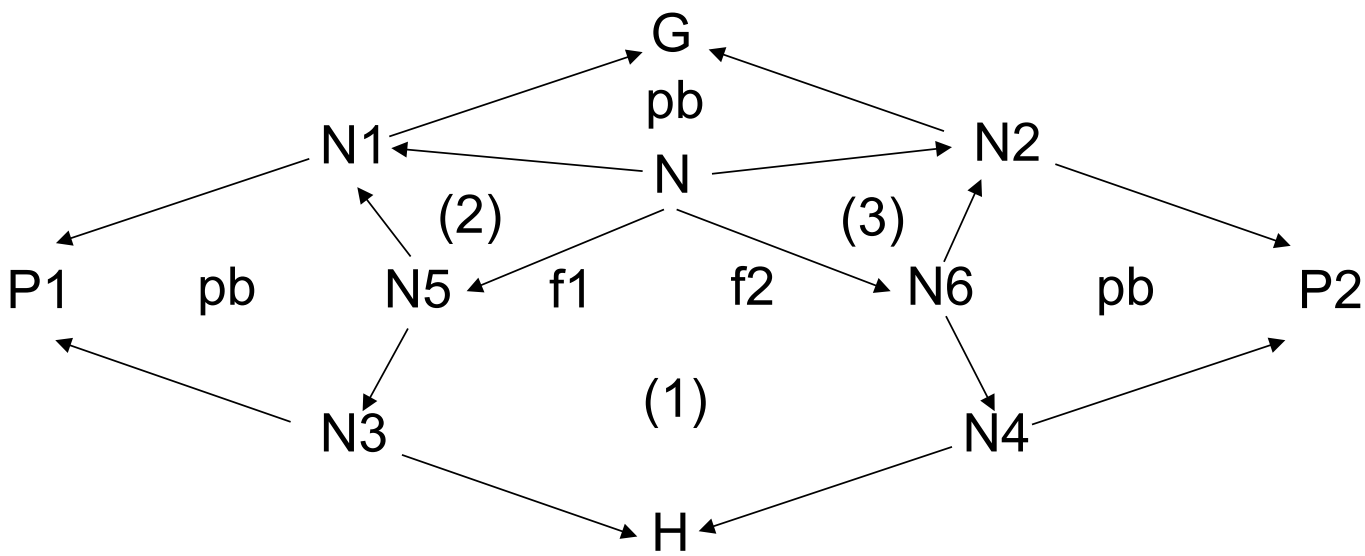
$$G1 \;{}_{p1,m1'}\!\Leftarrow G' \Rightarrow_{p2,m2'} G2$$

there is a critical pair $P1 \;{}_{p1,m1}\!\Leftarrow G \Rightarrow_{p2,m2} P2$ such that:



Strict Confluence

P1 $_{p1,m1}\Leftarrow$ G $\Rightarrow_{p2,m2}$ P2 is strictly confluent if there are morphisms f1 and f2 such that (1), (2), and (3) commute.



Local Confluence Theorem

A graph transformation system is locally confluent if all its critical pairs are strictly confluent.

¡Thank You!